DATA 100 Fall 2020 Midterm-Exam

INSTRUCTIONS

This is your exam. Complete it either at exam.cs61a.org or, if that doesn't work, by emailing course staff with your solutions before the exam deadline.

This exam is intended for the student with email address `<EMAILADDRESS>`. If this is not your email address, notify course staff immediately, as each exam is different. Do not distribute this exam PDF even after the exam ends, as some students may be taking the exam in a different time zone.

For questions with **circular bubbles**, you should select exactly *one* choice.

◯ You must choose either this option

◯ Or this one, but not both!

For questions with **square checkboxes**, you may select *multiple* choices.

☐ You could select this choice.

☐ You could select this one too!

You may start your exam now. Your exam is due at <DEADLINE> Pacific Time. Go to the next page to begin.

Preliminaries

You can complete and submit these questions before the exam starts.

(a) What is your full name?

(b) What is your Berkeley email?

(c) What is your student ID number?

(d) When are you taking this exam?

- ◯ Thursday 7pm PDT
- ◯ Friday 8am PDT
- ◯ Other

(e) Honor Code: *All work on this exam is my own.*

By writing your full name below, you are agreeing to this code:

(f) Important: You must copy the following statement exactly into the box below. Failure to do so may result in points deducted on the exam.

"I certify that all work on this exam is my own. I acknowledge that collaboration of any kind is forbidden, and that I will face severe penalties if I am caught, including at minimum, harsh penalties to my grade and a letter sent to the Center for Student Conduct."

1. (a) (8.0 points)

The Japanese Government would like to survey its 130 million citizens as part of its census. There are 125 million people currently in Japan, and 120 million of them are Japanese citizens. Somehow, the Japanese Government is able to survey all 125 million people currently in Japan, but isn't able to survey anyone outside of Japan. (Note that this is **not** a probability sample.) Everyone that the government surveys responds. For this question, terms like Japanese, American, Indian, etc. refer to people's citizenships, and we assume there is no dual citizenship.

We begin by categorizing the people we will and will not be surveying. For this question, each letter corresponds to the following category:

A: In the sampling frame and part of the population of interest.

B: In the sampling frame but *not* part of the population of interest.

C: *Not* in the sampling frame but part of the population of interest.

D: *Not* in the sampling frame and *not* part of the population of interest.

E: Insufficient information.

(2.0 points)

Choose the appropriate category for each individual listed below.

i. A. (0.5 pt) Japanese Prime Minister Yoshihide Suga, who is currently in Japan.

○ A

○ B

○ C

○ D

○ E

B. (0.5 pt) A Canadian tourist visiting Japan.

○ A

○ B

○ C

○ D

○ E

C. (0.5 pt) A Japanese businesswoman who has recently returned to Japan after a 10 year trip.

○ A

○ B

○ C

○ D

○ E

D. (0.5 pt) The Japanese Ambassador to Peru, situated in its capital Lima.

- ◯ A
- ◯ B
- ◯ C
- ◯ D
- ◯ E

(2.0 pt) Select all possible sources of bias and error in this method of surveying. (Remember our assumptions at the start.)

- ☐ ii. Selection bias
- ☐ Response bias
- ☐ Non-response bias
- ☐ Random error
- ☐ None of the above

iii. **(2.0 pt)** Suppose we select a single member of the sampling frame uniformly at random. What is the probability they are not in the population of interest?

- ○ 0
- ○ 1/13
- ○ 1/24
- ○ 1/25
- ○ 1
- ○ None of the above

iv. **(2.0 pt)** Suppose we select a single member of the population of interest uniformly at random. What is the probability that they are not in the sampling frame?

- ○ 0
- ○ 1/13
- ○ 2/13
- ○ 2/25
- ○ 1
- ○ None of the above

(8.0 points)

The Japanese Government would like to survey its 125 million citizens as part of its census. There are 120 million people currently in Japan, and 115 million of them are Japanese citizens. Somehow, the Japanese Government is able to survey all 120 million people currently in Japan, but isn't able to survey anyone outside of Japan. (Note that this is **not** a probability sample.) Everyone that the government surveys responds. For this question, terms like Japanese, American, Indian, etc. refer to people's citizenships, and we assume there is no dual citizenship.

We begin by categorizing the people we will and will not be surveying. For this question, each letter corresponds to the following category:

A: In the sampling frame and part of the population of interest.

B: In the sampling frame but *not* part of the population of interest.

C: *Not* in the sampling frame but part of the population of interest.

D: *Not* in the sampling frame and *not* part of the population of interest.

E: Insufficient information.

(2.0 points)

Choose the appropriate category for each individual listed below.

(0.5 pt) Japanese Prime Minister Yoshihide Suga, who is currently in Japan.

(b) i. A. ○ A

○ B

○ C

○ D

○ E

B. (0.5 pt) A Canadian tourist visiting Japan.

○ A

○ B

○ C

○ D

○ E

C. (0.5 pt) A Japanese businesswoman who has recently returned to Japan after a 10 year trip.

○ A

○ B

○ C

○ D

○ E

D. **(0.5 pt)** The Japanese Ambassador to Peru, situated in its capital Lima.

- ◯ A
- ◯ B
- ◯ C
- ◯ D
- ◯ E

(2.0 pt) Select all possible sources of bias and error in this method of surveying. (Remember our assumptions at the start.)

- ☐ ii. Selection bias
- ☐ Response bias
- ☐ Non-response bias
- ☐ Random error
- ☐ None of the above

iii. (2.0 pt) Suppose we select a single member of the sampling frame uniformly at random. What is the probability they are not in the population of interest?

- ◯ 0
- ◯ 1/24
- ◯ 1/25
- ◯ 23/25
- ◯ 1
- ◯ None of the above

iv. (2.0 pt) Suppose we select a single member of the population of interest uniformly at random. What is the probability that they are not in the sampling frame?

- ◯ 0
- ◯ 1/13
- ◯ 1/24
- ◯ 2/25
- ◯ 1
- ◯ None of the above

2. (10.0 points)

Suppose you are a famous country music artist. Every time you release a song, it lands on the Spotify Top 200 chart ("the charts") with a probability of $\frac{2}{3}$, independent of all other factors.

For the first four parts of this question, assume that you choose to release 12 songs in 2020.

(a) (2.0 pt) What is the probability that exactly half of the songs you release end up on the charts in 2020? Select all that apply.

- ☐ $\frac{1}{2}$
- ☐ $\binom{12}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^6$
- ☐ $\binom{12}{6}\left(\frac{1}{3}\right)^6$
- ☐ $1 - \sum_{k=0}^{5}\binom{12}{k}\left(\frac{2}{3}\right)^k\left(\frac{1}{3}\right)^{12-k} - \sum_{k=7}^{12}\binom{12}{k}\left(\frac{2}{3}\right)^k\left(\frac{1}{3}\right)^{12-k}$
- ☐ $1 - \sum_{k=0}^{5}\binom{12}{k}\left(\frac{2}{3}\right)^k\left(\frac{1}{3}\right)^{12-k}$

(b) (2.0 pt) What is the expected number of songs that you release in 2020 that **will not** end up on the charts? Round your answer to the closest integer.

(c) (3.0 pt) In 2021, you somehow determine that each song you release will chart with probability $\frac{2}{7}$. How many songs should you release in 2021 such that the expected number of songs that end up on the charts in 2021 is the same as it is in 2020? Give your answer as an integer.

(d) (1.0 pt) Let Y be the random variable representing the number of songs you release in 2020 that end up on the charts (some number between 0 and 12), and let Z be the random variable representing the number of songs you release in 2021 that end up on the charts (some number between 0 and your answer to the previous part).

Which of the following is true about the relationship between Y and Z? Select all that apply.

- ☐ Y and Z are equal
- ☐ Y and Z are identically distributed
- ☐ None of the above

(e) (2.0 pt) Now suppose you decide to release n songs in 2020, where n is some even integer. Which of the following functions $f(n)$ correctly return the probability that **at least half** of the songs you release in 2020 end up on the charts?

○

$$f(n) = \binom{n}{\frac{n}{2}} \frac{2^{\frac{n}{2}}}{3^n}$$

○

$$f(n) = \sum_{k=\frac{n}{2}}^{n} \binom{n}{k} \frac{2^k}{3^n}$$

○

$$f(n) = 1 - \sum_{k=0}^{\frac{n}{2}} \binom{n}{k} \left(\frac{2}{3}\right)^k \left(\frac{1}{3}\right)^{n-k}$$

○

$$f(n) = \sum_{k=\frac{n}{2}+1}^{n} \binom{n}{k} \left(\frac{2}{3}\right)^k \left(\frac{1}{3}\right)^{n-k}$$

○ None of the above

3. (13.0 points)

The table `billionaires` contains information about the ten most wealthy individuals in year1 and year2. Note that an individual can appear twice in the table if they appeared in both the year1 and year2 lists.

`billionaires` table schema:

```
CREATE TABLE billionaires (
    namereplace TEXT, /* name of individual */
    year INT, /* corresponding year */
    netWorth FLOAT, /* net worth in billions */
    company TEXT, /* primary company */
)
```

Below are 8 random rows from the `billionaires` table:

namereplace	year	netWorth	company
Jeff Bezos	year2	113	Amazon
Bill Gates	year2	98	Microsoft
Bill Gates	year1	86	Microsoft
Bernard Arnault	year2	76	LVMH
Mark Zuckerberg	year2	54.7	Facebook
Jeff Bezos	year1	72.8	Amazon
Amancio Ortega	year1	71.3	Zara
Warren Buffett	year2	67.5	Berkshire Hathaway

(a) (4.0 points)

i. (1.0 pt) Write a SQL query that returns the `namereplace`, `year`, and `netWorth` for all entries that have a `netWorth` greater than or equal to num1 and less than num2. Only show the first 5 rows.

Your query **must** follow the following structure.

```
SELECT ___________(a)____________
FROM billionaires
_____________(b)_________________
______(c)_______;
```

What goes in blank (a)?

ii. (2.0 pt) What goes in blank (b)?

iii. (1.0 pt) What goes in blank (c)?

(b) (9.0 points)

i. (2.0 pt) Write a SQL query to find the names of all billionaires who appear in the table for both year1 and year2. Your returned table should also have a column, `avgNetWorth`, that contains the average of their year1 and year2 net worth. The billionaire(s) with the highest average net worth should be at the top of the table.

Your answer **must** follow the following structure. *Hint: You cannot use a `GROUP BY`. Instead, think about how you can calculate `avgNetWorth` in your `SELECT` statement. Also think about how joining the `billionaires` table with itself could help.*

```
SELECT ___________(a)_____________ AS avgNetWorth
FROM ____________(b)_____________
WHERE _____(c)______
AND _____________(d)_____________
ORDER BY ____(e)_____ DESC;
```

What goes in blank (a)?

ii. (2.0 pt) What goes in blank (b)?

iii. (2.0 pt) What goes in blank (c)?

iv. (2.0 pt) What goes in blank (d)?

v. (1.0 pt) What goes in blank (e)?

4. (24.0 points)

Throughout this question, we are dealing with pandas DataFrame and Series objects. All code for this question, where applicable, must be written in Python. You may assume that pandas has been imported as `pd`.

The DataFrame `eb_hikes` provided below shows different hikes in the East Bay.
The information contained in the DataFrame includes:

- Trail: trail name which is unique (string)
- Elevation Gain: total elevation gain (ft) of a trail (int)
- Length: length of trail in miles (float)
- Location: City, State where the trails are located (string)

The first four rows of `eb_hikes` are shown below.

	Trail	Elevation Gain	Length	Location
0	Lake Temescal Loop	95	1.1	Oakland, CA
1	Inspiration Point	452	4.0	Berkeley, CA
2	Fire Trails	1496	4.2	Berkeley, CA
3	Piedmont Park Loop	137	0.8	Piedmont, CA

(a) **(2.0 pt)** Since all the trails are located in the East Bay, we know that they are all located in California. Write a line of Pandas code that changes `eb_hikes['Location']` to only display the city in which the trail is located. (For instance, the first four elements of `eb_hikes['Location']` would be `['Oakland', 'Berkeley', 'Berkeley', 'Piedmont']`.)

(b) **(3.0 pt)** We like to go on hikes with large Elevation Gains, and we also like to go on hikes in cityname. Write a line of Pandas code to create a `Series` containing the names of the trails who satisfy at least one of the following conditions:

- They are located in cityname
- Their Elevation Gain is at least elevnum ft

Assign your result to the variable `varname`. You may assume that `eb_hikes['Location']` has already been modified according to the previous part.

(c) **(9.0 points)**

Now suppose we have another DataFrame, `user_hikes`, that shows trails people have hiked with the following information:

- User: unique user name (string)
- Trail: trail name (string)
- Rating: user rating of the trail out of 5 (float)
- Difficulty: difficulty of trail (Easy, Medium, Hard, Very Hard) (string)
- Time Taken: time taken to complete the hike in minutes (int)

Note that users may appear in `user_hikes` more than once, if they have gone on more than one hike.

	User	Trail	Rating	Difficulty	Time Taken
0	Bob Honey	Piedmont Park Loop	2	Easy	10
1	Susie Thomas	Fire Trails	4.3	Very Hard	255
2	Josh Loop	Piedmont Park Loop	4	Easy	20
3	Susie Thomas	Inspiration Point	3	Medium	95
4	Bob Honey	Fire Trails	4	Hard	150

Say we want to filter `user_hikes` to only show rows for trails with an average rating among all users of num1 or higher as well as a maximum time taken among all users of num2 minutes or lower. Fill in the blanks below to create this DataFrame. Make sure to sort the output DataFrame first by the rating and then by the time taken to hike the trail. Your result should have the same columns as `user_hikes`.

```
def best_short_hikes(df):
    short_hikes = ___(a)___
    best_hikes = ___(b)___
    return short_hikes and best_hikes

user_hikes.____(c)____.filter(best_short_hikes).sort_values(____(d)____)
```

i. **(2.0 pt)** What goes in blank (a)?

ii. (2.0 pt) What goes in blank (b)?

iii. (2.0 pt) What goes in blank (c)?

iv. (2.0 pt) What goes in blank (d)?

v. (1.0 pt) Why might Bob Honey's Piedmont Park Loop rating still be in the output DataFrame you specified above even though his rating is less than num1 for that hike?

- ○ The time taken for his Piedmont Park Loop hike was less than num2 minutes so we don't care that his rating was less than num1.
- ○ Even with his rating of 2, the Piedmont Park Loop had an average rating of num1 or higher.
- ○ Bob Honey hiked the Fire Trails which had a rating of 4.
- ○ None of the above

(d) **(1.0 pt)** What type of variable is Difficulty in the `user_hikes` DataFrame?

- ◯ Quantitative discrete
- ◯ Quantitative continuous
- ◯ Qualitative nominal
- ◯ Qualitative ordinal

(e) **(7.0 points)**

As a reminder, here are the DataFrames `eb_hikes` and `user_hikes`, respectively:

`eb_hikes`

	Trail	Elevation Gain	Length	Location
0	Lake Temescal Loop	95	1.1	Oakland, CA
1	Inspiration Point	452	4.0	Berkeley, CA
2	Fire Trails	1496	4.2	Berkeley, CA
3	Piedmont Park Loop	137	0.8	Piedmont, CA

`user_hikes`

	User	Trail	Rating	Difficulty	Time Taken
0	Bob Honey	Piedmont Park Loop	2	Easy	10
1	Susie Thomas	Fire Trails	4.3	Very Hard	255
2	Josh Loop	Piedmont Park Loop	4	Easy	20
3	Susie Thomas	Inspiration Point	3	Medium	95
4	Bob Honey	Fire Trails	4	Hard	150

The DataFrame below shows the average minutes per mile that it takes each user to complete a trail.

Trail User	Fire Trails	Inspiration Point	Piedmont Park Loop
Bob Honey	35.714286	NaN	12.5
Josh Loop	NaN	NaN	25.0
Susie Thomas	60.714286	23.75	NaN

```
temp = user_hikes.___(a)___(eb_hikes, ______b______)
temp['Mins per Mile'] = ______(c)______
df = temp.____________________(d)__________________________
```

Fill in the blanks such that we output the DataFrame `df` above.

i. (1.0 pt) What goes in blank (a)?

ii. (1.0 pt) What goes in blank (b)?

iii. (2.0 pt) What goes in blank (c)?

iv. (3.0 pt) What goes in blank (d)?

(f) (2.0 pt) Consider the DataFrame `df` you created in the previous part. Suppose someone told you that User “Michael James” was the fastest recorded individual, as measured by average minutes per mile, to finish the Piedmont Park Loop (with no ties). Which of the following is guaranteed to correctly determine Michael James’ average minutes per mile for the Inspiration Point trail? Select all that apply.

- ☐ `df.loc[df['Piedmont Park Loop'] == df['Piedmont Park Loop'].min(), 'Inspiration Point']`
- ☐ `df.loc[~(df['Piedmont Park Loop'] <= df['Piedmont Park Loop'].max()), 'Inspiration Point']`
- ☐ `df.sort_values('Piedmont Park Loop').loc[:, 'Inspiration Point'].iloc[0]`
- ☐ `df.sort_values('Piedmont Park Loop').loc[:, 'Inspiration Point'].loc[0]`
- ☐ None of the above

5. (8.0 points)

HTTP is a response-reply internet protocol and is used by web-browsers to request content from servers to display to the user. The first few lines of an HTTP request have the following format:

```
POST /fa20/syllabus HTTP/1.1
User-Agent: Mozilla/4.0 (compatible; MSIE5.01; Windows NT)
Host: ds100.org
```

The first line contains an HTTP "verb", usually `GET` or `POST`, the path on the host being requested (`/fa20/syllabus` above), and the HTTP version being used to send the request. The lines below that are HTTP request headers that define the required parameters so that the server can process the request. (There are typically more, but we've omitted them since they're not relevant to this problem.)

(a) (4.0 pt) In the string `extract_verb`, write a regular expression that extracts the HTTP verb from a request.

For example:

```
>>> request_1 = """POST /fa20/syllabus HTTP/1.1
User-Agent: Mozilla/4.0 (compatible; MSIE5.01; Windows NT)
Host: ds100.org"""
>>> re.findall(extract_verb, request_1)[0]
'POST'

>>> request_2 = """GET /su19/syllabus HTTP/1.1
User-Agent: Safari/13.1 (Macintosh; Intel Mac OS X 10_10)
Host: data8.org"""
>>> re.findall(extract_verb, request_2)[0]
'GET'

>>> request_3 = """GARBAGE /useless HTTP/1.1
User-Agent: Garbage/0.0 (garbage)
Host: garbage.ca"""
>>> re.findall(extract_verb, request_3)[0]
'GARBAGE'
```

Your regex should work on requests that follow the format of the examples above. Assume that all HTTP requests have "HTTP" in them. Please write the regex as you would in Python with the form `extract_verb = r"..."`.

- Hint: Use capturing groups.
- Hint: Part of your regex may be the string `"\sHTTP/[\d.]+"`.

(b) (4.0 pt) In the string `extract_browser`, write a regular expression that extracts the name of the web browser (without its version) from a request.

For example:

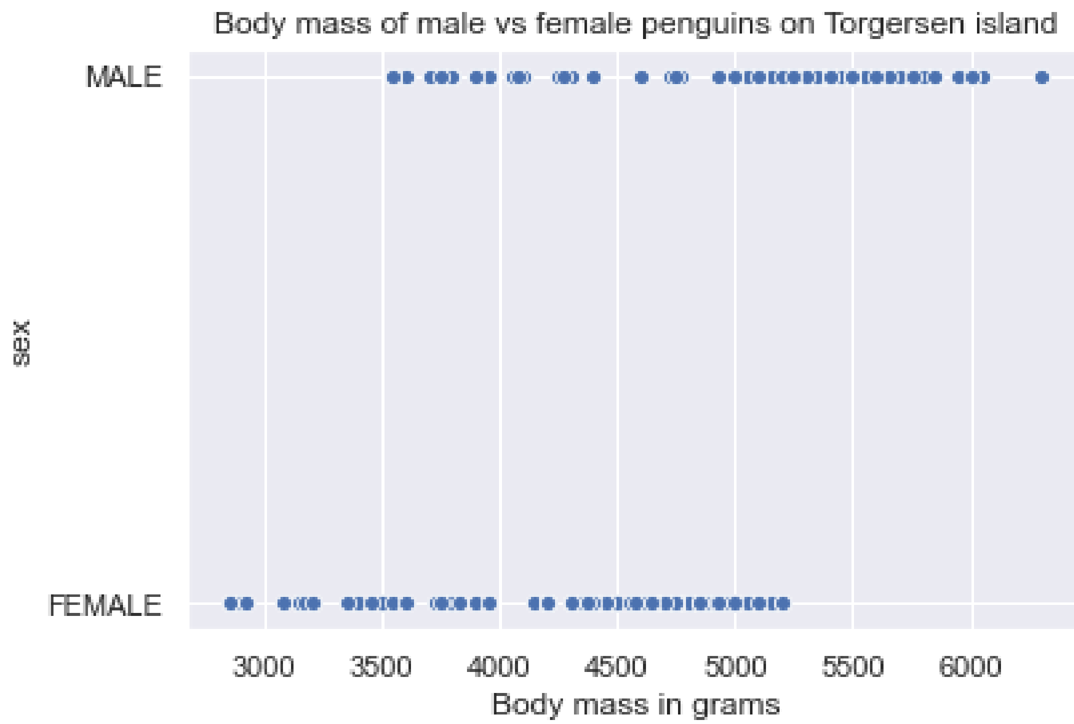
```
>>> request_1 = """POST /fa20/syllabus HTTP/1.1
User-Agent: Mozilla/4.0 (compatible; MSIE5.01; Windows NT)
Host: ds100.org"""
>>> re.findall(extract_browser, request_1)[0]
'Mozilla'

>>> request_2 = """GET /su19/syllabus HTTP/1.1
User-Agent: Safari/13.1 (Macintosh; Intel Mac OS X 10_10)
Host: data8.org"""
>>> re.findall(extract_browser, request_2)[0]
'Safari'

>>> request_3 = """GARBAGE /useless HTTP/1.1
User-Agent: Garbage/0.0 (garbage)
Host: garbage.ca"""
>>> re.findall(extract_browser, request_3)[0]
'Garbage'
```

Again, please write the regex as you would in Python with the form `extract_browser = r"..."`.

6. (6.0 points)

Answer each of the following questions regarding visualizations of a dataset collected about penguin species in the Palmer Archipelago in Antarctica.

Plot of body mass of male vs female penguins:



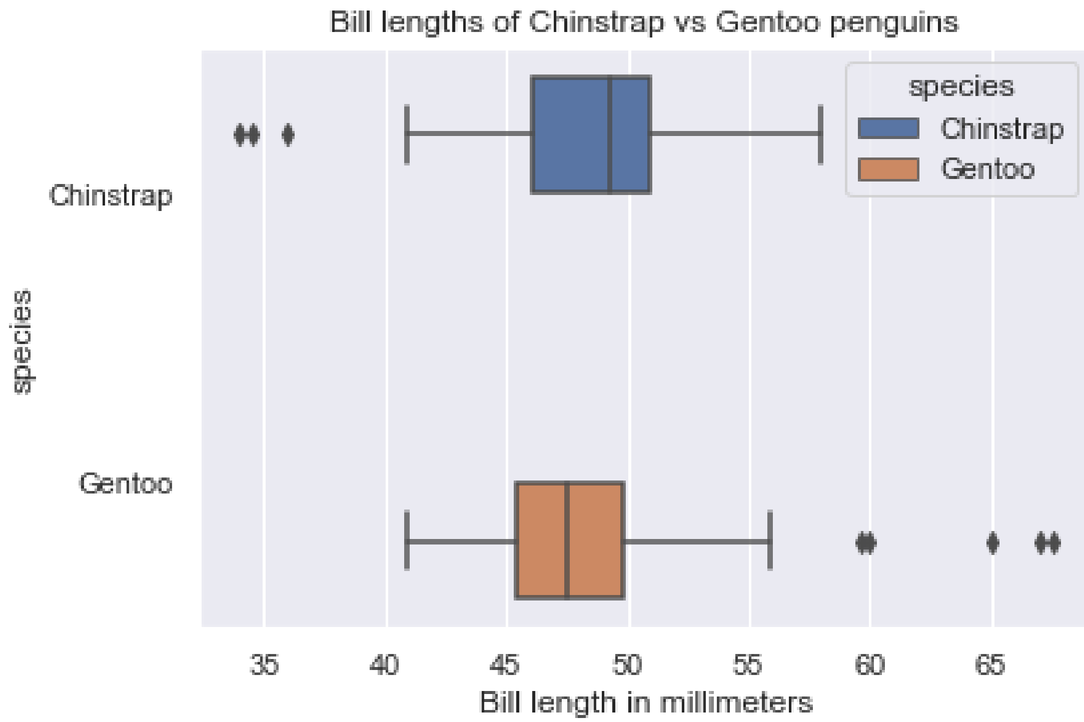
(a) (2.0 pt) The above visualization suffers from overplotting. Which of the following would be more appropriate types of visualization for this data? Select all that apply.

- ☐ Side-by-side line plots
- ☐ Overlaid density curves
- ☐ Side-by-side boxplots
- ☐ Bar chart

(b) (2.0 pt) Suppose instead we wanted to visualize bill length vs. body mass for all female penguins on the Archipelago. Which of the following would be appropriate types of visualization for this data? Select all that apply.

- ☐ Histograms
- ☐ 2D density curves
- ☐ Side-by-side boxplots
- ☐ Scatter plots

(c) (2.0 pt) Plot of bill lengths of Chinstrap vs Gentoo penguins:



We are now using a subset of the data, and visualizing the bill lengths of two penguin species using side-by-side box plots.

Which of these numbers is closest to the inter-quartile range of the Gentoo penguin bill lengths visualized above, in millimeters?

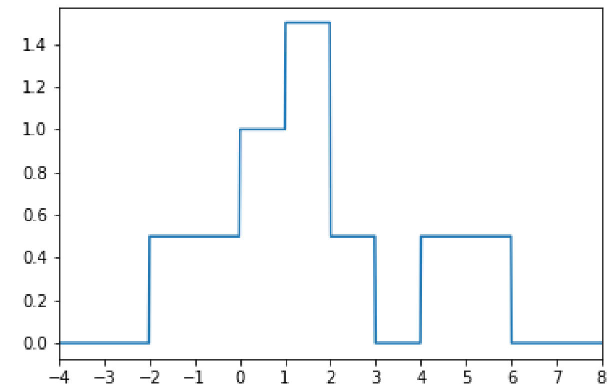
- ◯ 2.5
- ◯ 5
- ◯ 10
- ◯ 15

7. (5.0 points)

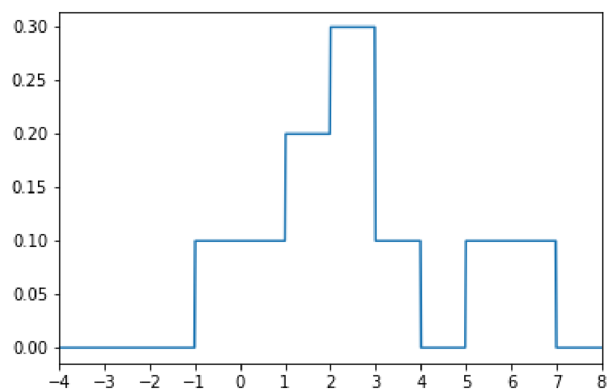
(a) (3.0 pt) For this question, suppose we have a dataset $X = [x_1, x_2, x_3, x_4, x_5] = [1, 1, -1, 5, 2]$.

Which of the following is the estimated density of X with the Boxcar kernel and bandwidth parameter $\alpha = 2$? Recall that the boxcar kernel is

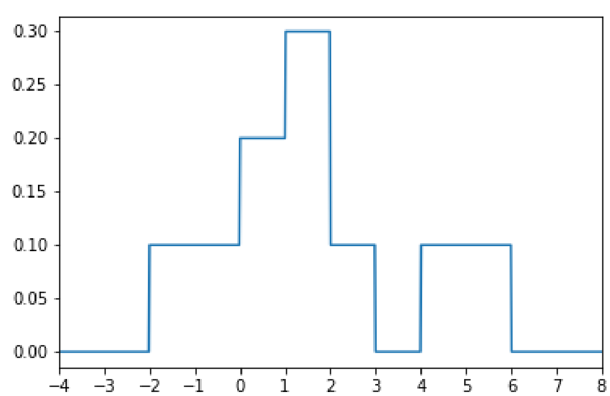
$$K_\alpha(x, x_i) = \begin{cases} \frac{1}{\alpha} & |x - x_i| \leq \frac{\alpha}{2} \\ 0 & \text{otherwise} \end{cases}$$



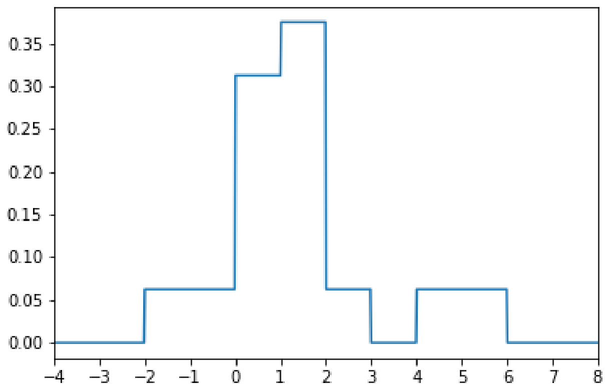
○



○

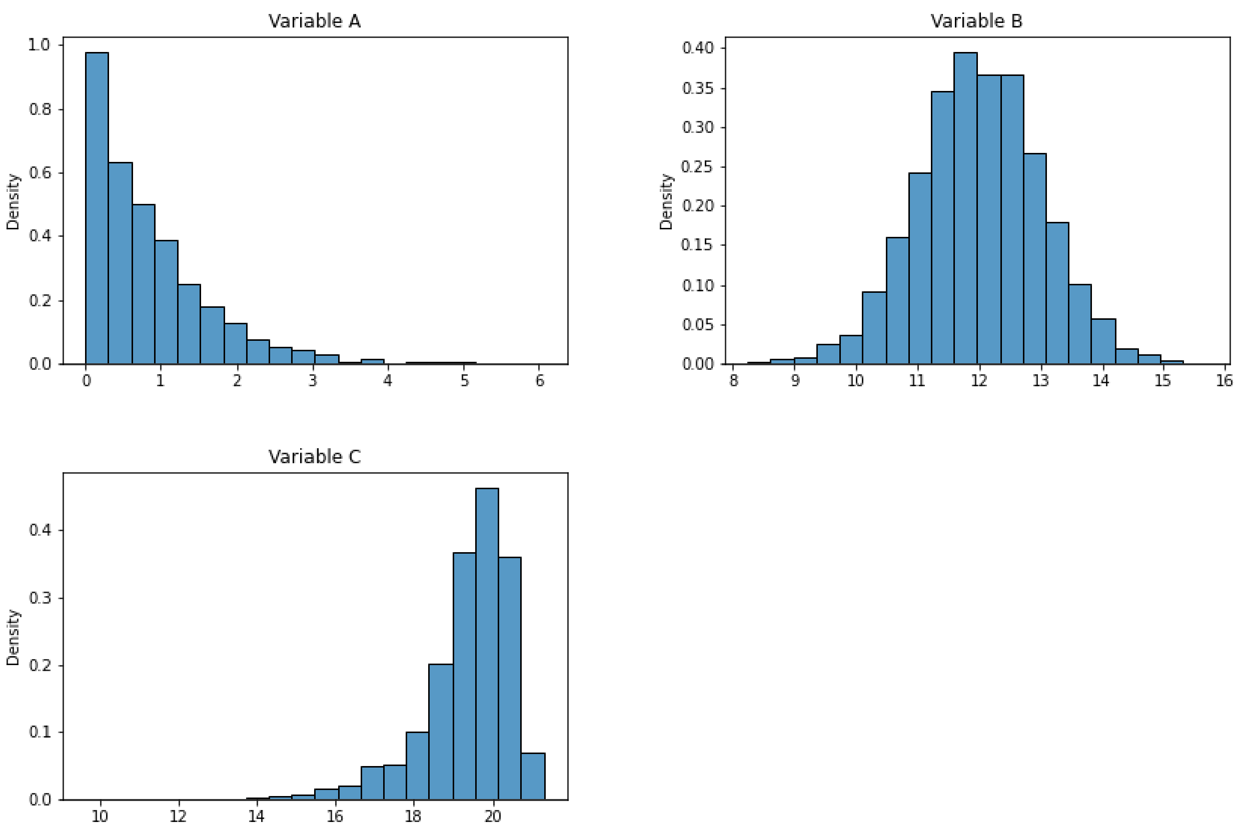


○



○

(b) (2.0 pt) Below, we show the distributions of three variables.



Which of the above distributions would be made more symmetric by applying a log transformation to the x-axis? Select all that apply.

☐ Variable A

☐ Variable B

☐ Variable C

☐ None of the above

8. (14.0 points)

Suppose we want to fit a constant model, $\hat{y} = \theta$, to a dataset $\{y_1, y_2, ..., y_n\}$.

(a) (3.0 pt) For the first two parts of this question, suppose we use the following loss function:

We choose to use the following loss function:

$$L(y_i, \theta) = -\ln\big(\frac{1}{\theta}\exp\big(-\frac{|y_i|}{\theta}\big)\big)$$

What is the derivative of the average loss R (i.e. empirical risk) for this choice of model and loss function with respect to θ?

- Hint: $\exp(x) = e^x$, and $\ln(x) = \log_e(x)$.
- Hint: $\ln(a\exp(b)) = \ln(a) + b$.

○ $\frac{dR}{d\theta} = 0$

○ $\frac{dR}{d\theta} = \frac{1}{n}\sum_{i=1}^{n}(\frac{1}{\theta} - \frac{|y_i|}{\theta^2})$

○ $\frac{dR}{d\theta} = \frac{1}{n}\sum_{i=1}^{n}(\frac{1}{\theta} + \frac{|y_i|}{\theta^2})$

○ $\frac{dR}{d\theta} = -\frac{1}{n}\sum |y_i|\exp(-\frac{|y_i|}{\theta})$

○ $\frac{dR}{d\theta} = \frac{1}{n}\sum \frac{|y_i|}{\theta}\exp(-\frac{|y_i|}{\theta})$

(b) (3.0 pt) Determine the value of $\hat{\theta}$ that minimizes average loss for the above choice of model and loss function.

○ $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} y_i$

○ $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} y_i^2$

○ $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} |y_i|$

○ $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} e^{|y_i|}$

○ $\hat{\theta} = \frac{1}{n}\frac{1}{\sum_{i=1}^{n}|y_i|}$

○ $\hat{\theta} = \frac{1}{n}\frac{1}{\sum_{i=1}^{n} y_i}$

(c) (2.0 pt) For the remaining parts of this question, consider the following two loss functions.

$$L_A(y_i, \theta) = 2(y_i - \theta)^2$$

$$L_B(y_i, \theta) = 3|y_i - \theta|$$

Let $\hat{\theta_A}$ and $\hat{\theta_B}$ be the values of θ that minimize average L_A loss and average L_B loss, respectively, for the constant model and a given dataset.

Which of the following is true regarding the relationship between $\hat{\theta_A}$ and $\hat{\theta_B}$ for the dataset $y = [1, 2, 3, 4, 5]$?

○ $\hat{\theta_A} > \hat{\theta_B}$

○ $\hat{\theta_A} = \hat{\theta_B}$

○ $\hat{\theta_A} < \hat{\theta_B}$

○ Impossible to tell

(d) **(2.0 pt)** Select the correct pair of optimal θ values for the dataset $y = [2, 2, 2, 2, 3, 3]$.

○ $\hat{\theta_A} = \frac{14}{3}, \hat{\theta_B} = 6$

○ $\hat{\theta_A} = \frac{7}{6}, \hat{\theta_B} = 2$

○ $\hat{\theta_A} = \frac{7}{6}, \hat{\theta_B} = \frac{2}{3}$

○ $\hat{\theta_A} = \frac{7}{3}, \hat{\theta_B} = 2$

○ None of the above

(e) **(2.0 pt)** Consider the dataset $y = [1, 2, 3, 4, c]$, where c is some constant greater than 4. As $c \to \infty$, what happens to $\hat{\theta_A}$ and $\hat{\theta_B}$?

○ $\hat{\theta_A}$ and $\hat{\theta_B}$ both increase

○ $\hat{\theta_A}$ increases, but $\hat{\theta_B}$ remains constant

○ $\hat{\theta_A}$ remains constant, but $\hat{\theta_B}$ increases

○ Both $\hat{\theta_A}$ and $\hat{\theta_B}$ remain constant

(f) **(2.0 pt)** Now consider another new loss function, $L_C(y_i, \theta) = (y_i - 2\theta)^2$.

True or False: For every dataset consisting of positive real numbers $y = [y_1, y_2, ..., y_n]$, the value of θ that minimizes average L_A loss is equal to the value of θ that minimizes average L_C loss (in other words, $\hat{\theta_A} = \hat{\theta_C}$).

○ True

○ False

9. (12.0 points)

(a) (2.0 pt) Consider the simple linear regression model $\hat{y} = \theta_0 + \theta_1 x$.

Which of the following expressions evaluate to $\hat{\theta_1}$, the value of θ_1 that minimizes average squared loss for the simple linear regression model? (Note, r is the correlation coefficient. You can assume $\hat{\theta_0}$ is already defined, and that $\bar{x}, \bar{y}, \sigma_x, \sigma_y \neq 0$.)

○ $\hat{\theta_1} = \frac{r}{\sigma_x^2}$

○ $\hat{\theta_1} = \bar{y} - \hat{\theta_0}\bar{x}$

○ $\hat{\theta_1} = \frac{\bar{y} - \hat{\theta_0}}{\bar{x}}$

○ $\hat{\theta_1} = \frac{1}{n\sigma_x\sigma_y}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$

(b) (3.0 pt) Now consider two models:

- Model A: $\hat{y} = \theta_0 + \theta_1 x$
- Model B: $\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2$

We fit both models using the same dataset, and we fit both models using all data available to us. The x_1 feature in the second model is the same as the x feature in the first model. As usual, we determine optimal coefficients by minimizing average squared loss.

Let RMSE_A and RMSE_B represent the root mean squared error on our dataset for Model A and Model B, respectively. Furthermore, let R_A^2 and R_B^2 represent the Multiple R^2 coefficient values for Model A and Model B, respectively. Lastly, let $\hat{\theta_{1,A}}$ and $\hat{\theta_{1,B}}$ represent the optimal values of $\hat{\theta_1}$ for Model A and Model B, respectively.

Which of the following statements are guaranteed to be true? Select all that apply.

☐ $\text{RMSE}_A \geq \text{RMSE}_B$

☐ $\text{RMSE}_A \leq \text{RMSE}_B$

☐ $R_A^2 \geq R_B^2$

☐ $R_A^2 \leq R_B^2$

☐ $\hat{\theta_{1,A}} = \hat{\theta_{1,B}}$

☐ $\hat{\theta_{1,A}} \neq \hat{\theta_{1,B}}$

☐ None of the above

(c) (3.0 pt) For the remainder of this question, we will use the multiple linear regression model, which is of the form

$$\hat{y} = x \cdot \theta = \sum_{j=0}^{p} \theta_j x_j$$

As in class, assume:

- $\mathbb{Y}$ is a vector containing our observed responses (i.e. true y's).
- $\mathbb{X}$ is a design matrix whose first column is 1 (i.e. $x_0 = 1$ for all observations), and $\mathbb{X}_i$ represents the ith row of $\mathbb{X}$.
- We determine $\hat{\theta}$ by minimizing average squared loss.

$\hat{\theta}$ is the minimizer of which of the following quantities? Select all that apply.

☐ $\sum_{i=1}^{n}(y_i - \theta)^2$

☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - \mathbb{X}_i \cdot \theta)^2$

☐ $\sum_{i=1}^{n}(y_i - \mathbb{X}_i \cdot \theta)^2$

☐ $\frac{1}{n}||\mathbb{Y} - \mathbb{X}\theta||_2^2$

☐ $||\mathbb{Y} - \mathbb{X}\theta||_2^2$

☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - \mathbb{X}_i \cdot \theta)$

☐ $\sum_{i=1}^{n}(y_i - \mathbb{X}_i \cdot \theta)$

☐ None of the above

(d) (2.0 pt) Suppose we have $n = 4$ observations. What are possible valid values for the residual vector $e = \mathbb{Y} - \mathbb{X}\hat{\theta}$ given our model and $\hat{\theta}$? Select all that apply.

☐ $e = [-1, 2, 3, -4]^T$

☐ $e = [-1, 2, 3, 4]^T$

☐ $e = [1, -1, 1, -1]^T$

☐ $e = [0, 0.1, 0, 0.1]$

☐ None of the above

(e) (2.0 pt) For this part, assume we have a design matrix and true response vector defined as follows:

$$\mathbb{X} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 4 \\ 1 & 2 & 0 \end{bmatrix}, \mathbb{Y} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$$

Suppose $\hat{\theta} = [-1, 4, 1]^T$. What is the squared loss for the second row in our dataset? Give your answer as an integer.

No more questions.